Dynamical screening of a polar optical phonon bound to a quantum well. Localised phonon-magnetoplasmon modes

1990 J. Phys.: Condens. Matter 2 6323

(http://iopscience.iop.org/0953-8984/2/29/010)

J. Phys.: Condens. Matter **2** (1990) 6323–6326. Printed in the UK

LETTER TO THE EDITOR

Dynamical screening of a polar optical phonon bound to a quantum well. Localised phonon–magnetoplasmon modes

Vadim L Gurevich and Kirill E Shtengel
A F Ioffe Institute of the Academy of Sciences of the USSR, 194021 Leningrad, USSR

Received 21 February 1990, in final form 18 May 1990

Abstract. We show that electrostatic interaction between bulk optical phonons and 2D plasmons leads to the formation of a coupled plasmon–phonon excitation localised near the plane of a quantum well. The dispersion of the coupled phonon–magnetoplasmon mode as well as the electric field distribution are determined.

During recent years various properties of quantum wells in semiconductors have been the subject of numerous experimental and theoretical investigations. Knowledge of details of dynamical screening of polar optical phonons by 2DEG in a quantum well is of basic interest for understanding many physical phenomena, such as (magneto-) transport.

As a result of dynamical screening, coupled (magneto-) plasmon–phonon modes appear. In this letter we show that interaction between bulk polar optical phonons and 2D plasmons leads to a new effect, i.e. localisation of the coupled phonon–plasmon excitations in the plane of the quantum well.

We consider a non-degenerate electron gas in a quantum well and are interested in the case where $k_B T < \Delta, \hbar\Omega_l < \Delta$ (Δ is the spacing between the first and the second size quantisation levels, Ω_l is the longitudinal optical phonon frequency). Besides this, we assume that all the lattice properties of the components of the heterostructure are the same. Hence, the phonons interact only with the electric field produced by the electrons of the quantum well. This means that we disregard another possible mechanism of localised phonon formation near a border between two different substances that have different dielectric and lattice properties, or confinement between two such borders (see [1–3]).

As for the electric field distribution, we obtain it both within the well and outside it. In this, our results differ from those obtained in [1, 4, 5] where only the dispersion relation was derived. The electric field distribution was also investigated in [6]. However, the approach used in [6] and based on the expression for the polarisation operator of a 3D electron gas can be justified only for the case where many sub-bands are involved.

The electrostatic potential φ of an excitation satisfies the self-consistent Poisson equation:

$$\Delta\varphi = -4\pi\,\delta\rho + 4\pi\,\mathrm{div}\,\boldsymbol{P} \tag{1}$$

or, in an equivalent form,

$$\varepsilon(\omega)\,\Delta\varphi = -4\pi\,\delta\rho. \tag{1a}$$

Here $\boldsymbol{P}$ is the polarisation of the ionic lattice, equal to $(\varepsilon(\omega)-1)\nabla\varphi/4\pi$:

$$\varepsilon(\omega) = \varepsilon_\infty(\omega^2 - \Omega_l^2)/(\omega^2 - \Omega_t^2) \tag{2}$$

(we neglect spatial dispersion of the ionic lattice polarisability); $\delta\rho$ is the induced charge density given by

$$\delta\rho(\boldsymbol{r}) = e^2\Pi(\omega, q)\,\langle 1|\phi(z)|1\rangle\,|\psi_1(z)|^2\exp[\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r}_\perp - \omega t)] \tag{3}$$

for the excitation potential in the form

$$\varphi(\boldsymbol{r}) = \phi(z)\exp[\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r}_\perp - \omega t)]$$

($\boldsymbol{q}$ and $\boldsymbol{r}_\perp$ are vectors in the plane of the quantum well), $\psi_1(z)$ is the eigenfunction of the first size quantisation level; $\Pi(\omega, q)$ is the 2D polarisation operator given (in the absence of magnetic field) by

$$\Pi(\omega, q) = \int \frac{n(\boldsymbol{k}-\boldsymbol{q}) - n(\boldsymbol{k})}{E(\boldsymbol{k}) - E(\boldsymbol{k}-\boldsymbol{q}) - \hbar\omega - \mathrm{i}0}\,\frac{2\,\mathrm{d}^2k}{(2\pi)^2}. \tag{4}$$

Here $E(\boldsymbol{k})$ and $n(\boldsymbol{k})$ are the energy and the occupancy factor of the electron state of the first size quantisation band with wave vector $\boldsymbol{k}$.

In a magnetic field $\boldsymbol{B}$ perpendicular to the plane of the quantum well, we have obtained for the polarisation operator (cf [7])

$$\Pi(\omega, q) = \frac{2n_s}{\hbar}\exp(-Q\,\mathrm{cotanh}\,\alpha)\sum_{N=1}^{\infty}\frac{N\omega_c}{N^2\omega^2 - \omega_c^2}\sinh(N\alpha)I_N\left(\frac{Q}{\sinh\alpha}\right). \tag{5}$$

Here $Q = q^2l^2/2$, $\alpha = \hbar\omega_c/2k_BT$, $l = \sqrt{c\hbar/eB}$ is the magnetic length, n_s is the surface electron density, I_N is the Bessel function of imaginary argument.

To find both the dispersion relation of the coupled plasmon–phonon mode and the spatial distribution of the electric field one should solve equation (1*a*) by inserting (3). The distribution of the field is given by

$$\phi(z) = \phi_0\int_{-\infty}^{+\infty}\mathrm{d}z'\,|\psi_1(z')|^2\exp(-q|z-z'|). \tag{6}$$

This mode is localised near the quantum well and its amplitude falls off exponentially

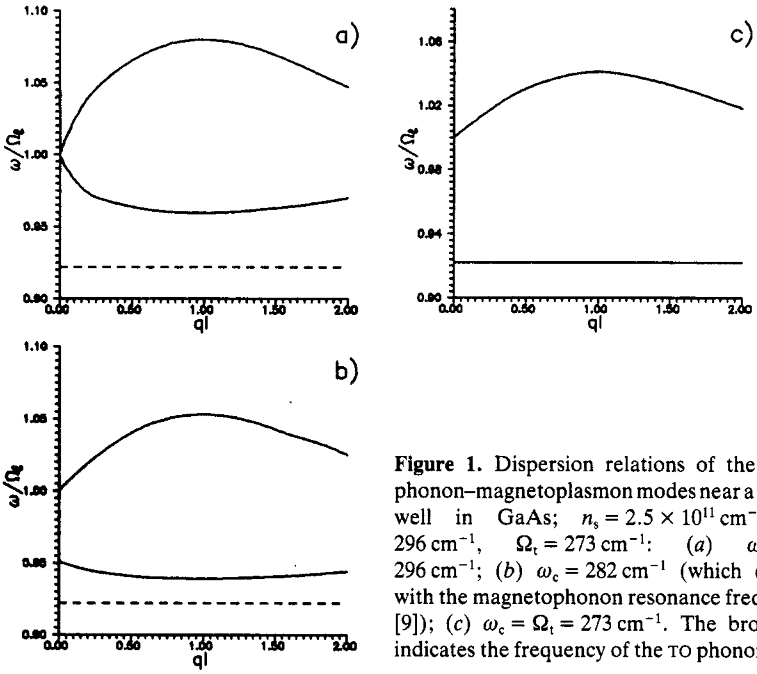

Figure 1. Dispersion relations of the coupled phonon–magnetoplasmon modes near a quantum well in GaAs; $n_s = 2.5 \times 10^{11}\,\mathrm{cm}^{-2}$, $\Omega_l = 296\,\mathrm{cm}^{-1}$, $\Omega_t = 273\,\mathrm{cm}^{-1}$: (*a*) $\omega_c = \Omega_l = 296\,\mathrm{cm}^{-1}$; (*b*) $\omega_c = 282\,\mathrm{cm}^{-1}$ (which coincides with the magnetophonon resonance frequency in [9]); (*c*) $\omega_c = \Omega_t = 273\,\mathrm{cm}^{-1}$. The broken line indicates the frequency of the TO phonon Ω_t.

outside the well with the decay length $1/q$. For the case of an infinitely deep rectangular quantum well, the electrostatic potential is

$$\varphi = \phi_0\, \mathrm{e}^{[\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r}_\perp - \omega t)]} \times \begin{cases} \left[\exp\left(\dfrac{qa}{2}\right)\left(1 + \dfrac{a^2q^2}{2\pi}\cos^2\dfrac{\pi}{a}z\right) - \cosh qz\right] & |z| \leqslant \dfrac{a}{2} \\ \sinh\left(\dfrac{qa}{2}\right)\exp\left[q\left(\dfrac{a}{2} - |z|\right)\right] & |z| > \dfrac{a}{2}. \end{cases} \tag{7}$$

The dispersion relation for such modes is

$$\varepsilon(\omega) + (2\pi e^2/q)\Pi(\omega, q) f(q) = 0 \tag{8}$$

where $f(q)$ is the form factor given by

$$f(q) = \int_{-\infty}^{+\infty} \mathrm{d}z\, |\psi_1(z)|^2 \int_{-\infty}^{+\infty} \mathrm{d}z'\, |\psi_1(z')|^2 \exp(-q|z - z'|). \tag{9}$$

In the absence of magnetic field, provided $qa < 1$ the dispersion relation is essentially the same as in [6]. (However, the spatial variation of the amplitude has not been discussed in [6].) In a strong magnetic field, magnetoplasmon–phonon modes exist with the same field distribution (2). We have obtained their dispersion relations for the case $qa < 1$, $|N\omega_c - \Omega_l| \ll \omega_c$, where $N = 1, 2, \ldots$; $\omega_c = eB/mc$:

$$\omega_\pm^2(q) = \frac{\Omega_l^2 + \omega_{\mathrm{mp}}^2}{2} \pm \left[\left(\frac{\Omega_l^2 - \omega_{\mathrm{mp}}^2}{2}\right)^2 + (\Omega_l^2 - \Omega_t^2)(\omega_{\mathrm{mp}}^2 - N^2\omega_c^2)\right]^{1/2} \tag{10}$$

where

$$\omega_{\rm mp}^2 = N^2\omega_c^2 + N\omega_c(4\pi e^2 n_s/\varepsilon_\infty \hbar q)\exp(-Q\,\mathrm{cotanh}\,\alpha)\sinh(N\alpha)\,I_N(Q/\sinh\alpha). \qquad (11)$$

Dispersion curves for localised magnetoplasmon–phonon modes are depicted in figure 1 (cf [4, 8] where the magnetoplasmon–phonon dispersion relation has been discussed for the case of degenerate statistics).

Obviously, our conclusion concerning the existence of optical phonons localised near the plane of a quantum well is valid only if the electron concentration is not too small. Otherwise the frequency of the localised mode appears to be too close to the value Ω_l and one should take into acocunt the dependence of the dielectric susceptibility ε not only on the frequency ω but also on the wave vector $\boldsymbol{q}$, i.e. its spatial dispersion.

The corresponding limitation imposed on the concentration for our theory to be valid is

$$n_s > n_c \equiv \varepsilon_c m\Omega_l^2 a_0^2/e^2 a \qquad (12)$$

for the case $qv_T < \Omega_l$. Here a_0 is the lattice constant while $1/\varepsilon_c = 1/\varepsilon_\infty - 1/\varepsilon_0$, and $v_T = \sqrt{2k_BT/m}$ is the thermal velocity. For typical quantum wells ($a \simeq 100$ Å), n_c is of the order of 10^{10} cm^{-2}.

Thus one might conclude that the dynamical screening can explain the shift of the magnetophonon resonance maxima positions observed in [9]. However, to come to a definite conclusion one should compare the strength of the interaction discussed with that for the bulk phonons, again taking into account the dynamical screening.

The authors wish to express their sincere gratitude to Yu M Galperin, E L Ivchenko, V I Kozub and G E Pikus for many valuable discussions.

References

[1] Bechstedt F and Enderlein R 1985 *Phys. Status Solidi* b **131** 53
[2] Babiker M 1986 *J. Phys. C: Solid State Phys.* **19** 683
[3] Mori N and Ando T 1989 *Phys. Rev.* B **40** 6175
[4] Oji H C A and MacDonald A H 1986 *Phys. Rev.* B **34** 1371
[5] Sato H and Hori Y 1989 *Phys. Rev.* B **39** 10192
[6] Wu Xiaoguang, Peeters F M and Devreese J T 1985 *Phys. Rev.* B **32** 6982
[7] Firsov Yu A and Gurevich V L 1962 *Sov. Phys.–JETP* **14** 367
[8] Wu Xiaoguang 1988 *Phys. Rev.* B **38** 4212
[9] Brummel M A, Nicholas R J, Hopkins M A, Harris J J and Foxon C T 1987 *Phys. Rev. Lett.* **58** 77